Problem Set:
Chapter 1: Introduction and a Brief Review of Relevant Tools

By JOAN LLULL*

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General instructions and hints

- ***Delivery instructions:*** The problem set is due by **Thursday, September 21st**. You should send it to me by email, including all codes you write. You can work either individually or in teams of up to three people.
- ***Programming instructions:*** You are asked to solve this problem set **twice**, a first time using Stata (or Matlab) pre-programmed routines —you have hints on the commands to use below—, and a second time with your preferred software without using any pre-programmed routine. In this second case, likelihood maximization/GMM minimization has to be programmed using the Newton-Raphson algorithm (try your best to program the N-R algorithm, but if you really struggle with it, I prefer you do the non-preprogrammed version of the problem set using build-in optimization routines and programming the rest). Write your code in a clear way, including comments that indicate what you do, so that I understand the code.
- ***Standard errors:*** Report standard errors for your estimates (compute them through the asymptotic formula for the variance of the estimator).
- ***Conditional/joint normality:*** Remember that if two variables are jointly normal $(x_1, x_2) \sim \mathcal{N}((\mu_1, \mu_2)', \Sigma)$, where Σ is formed of the variances σ_1^2 and σ_2^2 and the covariance $\rho\sigma_1\sigma_2$, then $x_1|x_2 \sim \mathcal{N}\left(\mu_1 + \frac{\sigma_1}{\sigma_2}\rho(x_2 - \mu_2), (1-\rho^2)\sigma_1^2\right)$.
- ***Parameter restrictions:*** Note that some parameters have restrictions. For example, variances have to be positive, and correlation coefficient should lie between zero and one. One easy way to implement this is by a change of variable: defining θ^* as the moving parameter, free to move in $(-\infty, \infty)$, we can express $\theta = \exp(\theta^*)/[1 + \exp(\theta^*)](b - a) + a$ to obtain a parameter in the domain (a, b), and, taking the limit to this expression, $\theta = \exp(\theta^*)$ to obtain a positive parameter .

* Departament d'Economia i Història Econòmica. Universitat Autònoma de Barcelona. Facultat d'Economia, Edifici B, Campus de Bellaterra, 08193, Cerdanyola del Vallès, Barcelona (Spain). E-mail: joan.llull[at]movebarcelona[dot]eu. URL: http://pareto.uab.cat/jllull.

- ***Stata hints:*** When solving the PS the first time you will need the commands `ml` and `gmm`. Stata help is quite useful; type `help` *command* to access it. There are tons of helpful resources on the web; here you have some:
  - http://www.ssc.wisc.edu/sscc/pubs/stata_prog1.htm: A gentle introduction to programming in Stata.
  - http://www.stata.com/manuals13/u18.pdf: Detailed manual on programming in Stata
  - http://kumlai.free.fr/RESEARCH/THESE/TEXTE/STATA/MLE_in_Stata.pdf: Step-by-step for programming a MLE using `ml` Stata command.
  - http://www.stata.com/meeting/germany10/germany10_drukker.pdf: Introduction to using GMM with Stata.

General setup and introduction

Consider the following model for the wage determination of married women:

$$\begin{aligned} lwage_i &= \beta_0 + \beta_1 educ_i + \beta_2 exper_i + \beta_3 exper_i^2 + \varepsilon_i \\ educ_i &= \gamma_0 + \gamma_1 exper + \gamma_2 exper^2 + \gamma_3 meduc + \gamma_4 feduc + \nu_i, \end{aligned}$$

with:

$$\begin{pmatrix} \varepsilon_i \\ \nu_i \end{pmatrix} \bigg| (exper_i, exper_i^2, meduc_i, feduc_i)' \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon\sigma_\nu \\ \rho\sigma_\varepsilon\sigma_\nu & \sigma_\nu^2 \end{bmatrix} \right),$$

where variable $lwage_i$ indicates the log of monthly earnings for individual i, $educ_i$ is the number of years of schooling, $exper_i$ and $exper_i^2$ are the number of years of experience and its square, $meduc_i$ is mother's education and $feduc_i$ is the education of the father. This kind of wage regressions are very popular in the empirical labor literature to estimate the returns to education, and are known as Mincer equations, after Mincer (1974).

In this problem set you are going to estimate this model (which is a standard instrumental variables one) by Maximum Likelihood and GMM. In the ML exercise you will also learn that the standard ML estimator can be called *full information* ML (FIML), and that it can be approximated by an alternative estimator that proceeds with the estimation by breaking up the problem in different sub-parts. This alternative estimator is called *limited information* ML or LIML). This will come up quite useful later on in the course. In the GMM exercise, you will start by noting that the GMM estimation in this case can be implemented by matrix multiplication and then you will practice estimation minimizing the standard criterion function, which is what you would do in more nonlinear types of problems.

Exercise 1. FIML vs LIML estimation with an application for the estimation of wages of married women

Denote by Z the matrix that includes all observations for all variables but $lwage$ and $educ$, with $\boldsymbol{z}_i = (exper_i, exper_i^2, meduc_i, feduc_i)'$, and by $\boldsymbol{z}_i$ the vector that corresponds to each observation (each row in that matrix).

1) Write the conditional log-likelihood function for one observation given $\boldsymbol{z}_i$. [***Hint:*** *factorize the joint conditional likelihood of* $lwage_i$ *and* $educ_i$ *given* $\boldsymbol{z}_i$ *into the product of a conditional density of* $lwage_i$ *given* $educ_i$ *and* $\boldsymbol{z}_i$*, and a "marginal" density of* $educ_i$ *given* $\boldsymbol{z}_i$*. See the general hint above on normal distributuions for help.*]

2) Using the data set included in the sheet Wages from the file *PS1.xls*, obtain the MLE of all parameters of the model. Respect the general instructions listed above!

3) Which parameters do the "marginal" and the conditional densities from question 1 depend on? Estimate the parameters of the "marginal" density of $educ_i$ given $\boldsymbol{z}_i$ using data on $educ$ given $\boldsymbol{z}$ only.

4) Now estimate the remaining parameters of the model using the conditional density of $lwage_i$ given $educ_i$ and $\boldsymbol{z}_i$ and the estimates of the parameters obtained in question 3.

5) The two-step estimation procedure described in questions 3-4 is known as *Limited Information Maximum Likelihood (LIML)* estimator, which is opposed to the general *Full Information Maximum Likelihood (FIML)* estimator from question 2. Do you think this procedure provides a consistent estimator of the parameters? Why? How do you think its asymptotic variance compares to the one of the FIML estimator?

Exercise 2. GMM estimation of wages of married women

1) Write the moment conditions that identify the model above. [***Hint:*** *only use moment conditions from the first moments.*]

2) Derive the estimator analytically, and implement it (through matrix multiplication) using the data set above (in the preprogrammed part of the problem set, replace that by estimation using stata commands).

3) Now re-estimate minimizing the objective function numerically.