Hicks-Marshall Conditions and Defining Antitrust Markets for Intermediate Goods

James Langenfeld, Jonathan T. Tomlin, David A. Weiskopf, and Georgi Giozov

December 22, 2008

DRAFT

Abstract

Appropriate definition of the relevant market for an input is often highly important in antitrust inquiries. We combine the well established “Hicks-Marshall” conditions of derived demand with critical loss/critical elasticity of demand analysis to yield important insights into the appropriate definition of antitrust markets for inputs. We show that solely examining demand substitution possibilities for direct consumers of an input can lead to the wrong market definition. We also demonstrate how critical loss analysis in conjunction with Hicks-Marshall conditions can provide greater insights for delineating relevant markets for inputs than apparent from some Federal Trade Commission decisions. Finally, we illustrate how reasonable ranges for key parameters relating to the ability of users of an input to substitute to other inputs and to downstream market conditions will often be sufficient to define antitrust markets in practice.

1 Langenfeld: LECG and Loyola University Chicago; Weiskopf: LECG and Johns Hopkins University; Tomlin and Giozov: LECG. The opinions expressed are those of the authors, and do not necessarily represent the views of any institution.
I) INTRODUCTION

“Intermediate goods” or “inputs” are ubiquitous in our economy. Not surprisingly, antitrust cases often concern mergers or allegedly anticompetitive practices that involve manufacturing intermediate goods and services. These inputs are typically not sold directly to the ultimate consumers, but instead are combined with other inputs to produce one or more “downstream” products. Inputs can differ substantially in the number of products or services in which they are used; their importance in the production of a downstream product or products; and the number of levels of production and distribution that exist between production of the input and sale of the downstream products incorporating that input. In some industries, part of the production of an input is sold to third party manufacturers (“merchant sales”) and part of the production is not sold but instead used by vertically integrated manufacturers to produce a downstream product.

The outcome of antitrust inquiries has often turned on the definition of the relevant markets and market power for intermediate goods. As has been established in the labor economics literature for decades, and recognized by some in the antitrust context, properly evaluating the demand for an input often involves examining market conditions downstream from that input. However, antitrust analysis has often differed in whether or not downstream conditions have been evaluated in assessing the market power question for inputs. This is unfortunate. Not only can downstream conditions help inform the market power question for inputs, failure to examine these conditions can lead to improper conclusions.

In this article, we demonstrate how the well established “Hicks-Marshall” conditions of derived demand can be used in conjunction with critical loss/critical elasticity of demand analysis to properly define antitrust markets for inputs. Our analyses have several important implications for economic evaluation of a merger or alleged anticompetitive actions. First, solely examining demand substitution of direct customers of an input, often the primary focus of antitrust relevant market inquiries, can lead to

---

wrong conclusions about input antitrust markets. Second, critical loss analysis using information on technical production substitutability and downstream market conditions can provide useful market definition analyses when direct estimates of the elasticity of demand of the input are unavailable. Third, using information on downstream market conditions can also often serve as a “reality check” on other market definition analyses. Finally, taking into consideration the Hicks-Marshall conditions, general information on the technical abilities to substitute and downstream market conditions can often be used to define relevant input markets in practice. Reasonable ranges on the key parameters, as opposed to precise estimation, will often be sufficient to define relevant markets for inputs in conjunction with critical loss/critical elasticity of demand analysis.

Section II below provides a brief summary of techniques for defining relevant markets, with particular emphasis on critical loss/critical elasticity of demand analysis and Hicks-Marshall conditions of derived demand. Section III provides examples of antitrust cases involving intermediate goods. Section IV combines the Hicks-Marshall conditions with critical loss/critical elasticity of demand analysis in the context of defining relevant markets for intermediate goods. As we show, downstream conditions can often provide useful information on relevant market definition for inputs, and overlooking these conditions can lead to improper antitrust market definitions. Section V summarizes our conclusions.

II) INTERMEDIATE GOOD MARKET DEFINITION AND HICKS-MARSHALL CONDITIONS

A. Market Definition

Market definition remains an important part of evaluating most competition matters both in the United States and the European Union.3 The economics of the U.S. Department of Justice and Federal Trade Commission Merger Guidelines’ (“Merger Guidelines”) approach for market definition focuses initially on buyers’ demand and the products or services over which a “hypothetical monopolist” could profitably raise prices.

3 A number of economists have questioned the need for it if there is direct evidence of anticompetitive effects from an alleged anticompetitive act. At a minimum, market definition analyses can be used to check the robustness of any analysis that attempts to measure the direct effects of an alleged anticompetitive act.
to customers. This approach asks whether a small but significant (usually five to ten percent) and non-transitory price increase over existing levels by all firms in the proposed market would be profitable. One key element of this approach is determining the degree to which customers would switch to other products if prices of the products in a proposed market were raised. If buyers can relatively easily substitute other products – i.e., there is a high elasticity of demand – then the hypothetical market needs to be expanded to take into account other substitute products. With regard to inputs, the Merger Guidelines state the applicable agency will take into account “the influence of downstream competition faced by buyers in their output markets.”

This approach to market definition can and is also used in non-merger settings, as is recognized by the European Union, although there can be issues in applying this type of test to monopolization and conspiracy cases.

As part of this market definition analysis, economists often consider indicia similar to those discussed by the Supreme Court in Brown Shoe Co. v. United States, including “industry or public recognition of … the product’s peculiar characteristics and uses including a consideration of functional and economic substitutability, unique production facilities, distinct customers, distinct prices, sensitivity to price changes, and specialized vendors.” This evidence typically comes from a variety of sources, and can be used to provide at least some general parameters for the factors that go into market definition.

---

4The Merger Guidelines define a relevant market as “a product or group of products and a geographic area in which it is produced or sold such that a hypothetical profit-maximizing firm … that was the only present and future producer or seller of those products in that area likely would impose at least a ‘small but significant and non-transitory’ increase in price.” Merger Guidelines, p. 4. The European Commission applies a very similar test for market definition. “17. The question to be answered is whether the parties’ customers would switch to readily available substitutes or to suppliers located elsewhere in response to a hypothetical small (in the range 5 % to 10 %) but permanent relative price increase in the products and areas being considered. If substitution were enough to make the price increase unprofitable because of the resulting loss of sales, additional substitutes and areas are included in the relevant market. This would be done until the set of products and geographical areas is such that small, permanent increases in relative prices would be profitable.” Commission Notice on the definition of relevant market for the purpose of the Community Competition Law, Official Journal C 372, 09/12/1997 P. 0005-0013.

5 Merger Guidelines, Section 1.11.


“Critical loss” analysis has been used to help define relevant markets in many intermediate goods antitrust cases, although its application has been somewhat controversial. Critical loss is a direct extension of the Merger Guidelines concept that an antitrust market is a product and an area in which prices can profitably be raised. The idea of the critical loss approach is to test whether a price hike might cause sales losses from all sources (sales lost to alternative products or to producers outside the given area) that were so great that the proposed price hike was unprofitable. The approach follows three steps: (1) estimating the hypothetical monopolist’s per unit variable profit margin before prices would be increased; (2) determining the percentage of sales this hypothetical monopolist could lose before a price increase becomes unprofitable; and (3) estimating whether this hypothetical monopolist would lose this percentage of sales if it increased price.

Assuming one can get reliable measures of per unit variable profit margins, a price increase will be profitable if and only if the total variable profit at the higher price exceeds the total variable profit at the lower price, which can be shown algebraically as:

\[(P + \Delta P - MC)(Q - \Delta Q) > (P - MC)Q\]  

where: 
- \(P\) is the initial price
- \(\Delta P\) is the price increase
- \(MC\) is the firm’s constant marginal cost
- \(Q\) is the initial quantity sold
- and \(\Delta Q\) is the decline in sales caused by the price increase.

Combining terms and rearranging yields.

\[\frac{\Delta P}{P + \Delta P - MC} = \frac{t}{m + t} > \frac{\Delta Q}{Q}\]  

Where \( t = \Delta P/P \) is the percentage price increase; \( m = (P - MC)/P \) is the initial margin; 
\[
\frac{t}{m + t}
\]
and \( m + t \) is the “break-even” critical loss. At the break-even critical loss for a given price increase, a hypothetical monopolist will have its profits unchanged by that price increase. For that price increase to increase profits, the hypothetical monopolist would have to lose less than this break-even critical loss. Accordingly, if the hypothetical monopolist would lose less than the critical loss, then this analysis indicates that the market should be no larger than that being analyzed.

An alternative way to implement this type of market definition analysis is to calculate a “critical elasticity of demand”. That is, critical loss can be expressed as the critical elasticity of demand facing the candidate antitrust market.\(^{11}\) Under this alternative approach, if the actual elasticity of demand is less (in absolute value) than the critical elasticity of demand, then the products constitute a relevant antitrust market.

The final step in this critical loss or critical demand analyses is then to estimate how many sales would be lost for a given price increase. This is often done through econometric estimates of the elasticity of demand, natural experiments,\(^{12}\) and/or qualitative information on the industry.

Some economists have advocated examining a hypothetical profit maximizing price increase, rather than strictly applying a 5 percent breakeven price increase test.\(^{13}\) It can be shown that the profit maximizing price change from the status quo is half of the break even price change under certain circumstances. Given these circumstances, it is profit maximizing to increase price by \((.5) \times t\) if it is more profitable than the status quo to increase price by \(t\).\(^{14}\) This suggests that a 10 percent price increase test for critical loss may be more appropriate than a 5 percent test.

\[^{12}\text{See Mary Coleman and James Langenfeld,} \textit{Natural Experiments,} \text{in Wayne D. Collins, ed.,} \textit{Issues in Competition Law and Policy,} \text{American Bar Association, 2008.}\]
\[^{14}\text{See, for example, Joseph Farrell & Carl Shapiro, Improving Critical Loss Analysis,} \textit{Antitrust Source,} \text{Feb. 2008, and the papers mentioned there.}\]
Economists have disagreed on the type of evidence that should be used to evaluate whether a price increase would be profitable, and whether the critical loss formula needs to be modified to reflect how the industry has been acting. In particular, some commentators have argued that one can infer an elasticity of demand under the assumption of profit maximization from the price-variable cost margin, known as the Lerner Index.\textsuperscript{15} That is, $\epsilon = 1/ m$, where $\epsilon$ is the elasticity of demand for the product or service in a candidate relevant market. The actual loss from a price increase based on this reasoning is approximately $\epsilon \times t$, or equivalently $t/m$. Farrell and Shapiro\textsuperscript{16} propose that one estimate the actual loss as $L = (1-A)(t/m)$, where $L$ is the actual loss due to a price increase of $t$, and $A$ is the Aggregate Diversion Ratio defined as the fraction of sales lost by one product or geographic area that go to other products in the candidate market when its price alone increases by $t$.\textsuperscript{17}

Finally, market definition analysis typically focuses on demand side substitutability for the immediate customers of the product at issue.\textsuperscript{18} As we show below, in the case of inputs, both demand side substitutability of the immediate customers of a product and the demand side substitutability of indirect customers purchasing “downstream” products incorporating that input can be important in defining a relevant antitrust market.

B. Hicks-Marshall Conditions

The demand for an input is a “derived demand” in that its demand is derived from the demand for the ultimate product or service for which it acts as an input. Absent demand for a downstream product, there would be no demand for the input. Market conditions “downstream” from an input help determine the elasticity of demand for that input and, therefore, the relevant market for the input.

\textsuperscript{16} Farrell and Shapiro, \textit{Id}.
\textsuperscript{17} Of course, estimating $A$ can often be more challenging than estimating $\epsilon$ directly from market data and other available information.
In labor economics, the elasticity of demand for an input has long been evaluated using conditions originally developed by Alfred Marshall\(^\text{19}\) and revised by John Hicks,\(^\text{20}\) commonly referred to as the “Hicks Marshall” conditions of derived demand. In their simplest form, these conditions evaluate the elasticity of demand for an input under the assumption of two inputs (typically capital and labor), constant returns to scale, and perfect competition downstream.\(^\text{21}\) These conditions state that, other things equal, the demand for an input is less elastic when:

a. *The elasticity of substitution between inputs is low.*

b. *The elasticity of demand for the downstream product is low.* If demand for the downstream product is inelastic, then any price increase caused by a higher price of the input will lead to only a small change in the quantity demanded of the downstream product and is likely to lead to only a small change in the quantity demanded of the input. Consideration of the elasticity of demand for the downstream product can be very important for defining a relevant market for an input.

c. *The cost of the input is a small share of the total cost of producing the downstream product.* This condition holds when the elasticity of substitution is less than the elasticity of demand for the downstream product.

d. *The supply of other inputs to production is less elastic.* When the elasticity of supply of other inputs is less elastic, substitution to those inputs in response to a price increase in an input is discouraged because the increased demand for those inputs causes their prices to increase significantly.\(^\text{22}\) As a practical matter, this condition is important only in those instances where substitution in response to the posited input price increase would be great enough to increase prices of other inputs significantly.

Formally, based on the Hicks-Marshall conditions, the elasticity of demand for an input can be expressed as a function of the elasticity of demand for the downstream

---


\(^{22}\) The Merger Guidelines generally assume that alternatives to a product are available in unlimited quantities at constant prices, an assumption that, if not true, tends to overstate the demand elasticity of an input.
product, the elasticity of substitution between the inputs, and the elasticity of supply of the second input as follows (with demand elasticities expressed in absolute value terms):  

\[ \varepsilon_i = \frac{E_i S_i + n_i [v_i E_f + (1 - v_i) S_i]}{n_i + [v_i S_i + (1 - v_i) E_f]} \]  

(1)

Where \( \varepsilon_i \) is the own price elasticity of demand for input \( i \), \( E_f \) is the own price elasticity of demand for the downstream product incorporating input \( i \), \( S_i \) is the elasticity of substitution between input \( i \) and the second input used in the production of downstream good \( f \), \( v_i \) is the proportion of total costs of downstream product \( f \) accounted for by input \( i \), and \( n_i \) is the elasticity of supply of the second input used to product downstream product \( f \).

Whether the Hicks-Marshall conditions can be rigorously applied in practice in defining relevant markets depends on the products or services at issue and availability of information. For example, Hicks-Marshall conditions may be difficult to apply when several different downstream products use the input, where each has a different elasticity of demand. They may also be difficult to apply when many other inputs are used in the production of the downstream product, and these inputs act as important substitutes and/or complements for the input being examined. The Hicks-Marshall conditions are easiest to apply when the input at issue is used primarily to produce a small number of downstream products, other inputs do not act as good substitutes for the input at issue, and the elasticity of supply of other inputs is high. For example, with a fixed factor production function, the elasticity of demand for the input is simply the proportion of total cost of the downstream product borne by that input multiplied by the elasticity of demand for the downstream product.

III) PREVIOUS ANTITRUST ANALYSIS OF INPUT MARKETS

Relevant market definition has been an issue for numerous intermediate goods such as coal, crude oil, lead, steel, and chemicals. However, antitrust analysis in

---

23 Layard and Walters, Id. at 267.
24 It may not be necessary to consider each of the Hicks-Marshall conditions if it is clear that the input at issue faces strong competition from other substitute inputs.
private litigation and by antitrust authorities has differed on whether or not explicit consideration has been given to downstream conditions in assessing relevant markets for inputs and addressing the critical issue of market power. Many court decisions address the relevant market for an input without any mention of downstream conditions. On the other hand, at least a few decisions demonstrate a consideration of downstream conditions.

In their analysis of two mergers involving the chemical polyvinyl chloride (“PVC”), the Federal Trade Commission (“FTC”) took into account three of the four “Hicks-Marshall” conditions of derived demand. PVC is a thermoplastic resin combined with other additives to produce finished vinyl products (such as pipe and siding). In evaluating the elasticity of demand for PVC in determining the competitive effects of the merger between B.F. Goodrich and Diamond Shamrock, the FTC noted that “price elasticities increase as (1) the degree to which other inputs can be substituted for them increases; (2) the proportion of total costs for which each accounts increases; and (3) the price elasticity of demand for PVC end products and PVC resin – the products for which they are respectively used – increases.” After assessing these three conditions, the FTC concluded that the PVC was characterized by inelastic demand. In later considering the elasticity of demand of PVC in the acquisition by Occidental Chemical Corporation (“Octel”) of the PVC business of Tenneco Polymers, the FTC also referred to these three

27 RSR Corp. v. FTC, 602 F.2d 1317, 1320-22 (9th. Cir. 1979).
Hicks-Marshall conditions (although, like in *Goodrich*, no mention is made of a fourth Hicks-Marshall condition – the elasticity of supply of other inputs).33

In *Associated Octel*, the FTC challenged a series of agreements among manufacturers of lead antiknock compounds for leaded gasoline.34 The FTC alleged that lead antiknock compounds constituted a relevant product market, and this decision relied upon the reasoning of the Hicks-Marshall conditions. First, the FTC found that the downstream demand for high octane fuels was relatively inelastic because consumers of high octane fuels could not easily switch to other, lower octane fuels, such as general automotive gasoline. Second, the cost of the additive was relatively small compared to total cost of the high octane fuels. Third, there was limited ability to substitute other types of inputs for the lead antiknock compounds in production. Refiners could not easily or inexpensively switch away from the lead compounds to other octane enhancing additives, such as methyl tertiary-butyl ether (MTBE).35 Determining whether the demand for lead antiknock compounds was sufficiently inelastic to constitute a relevant product market required a thorough understanding of the production technology of the immediate customers (the refiners) and demand by final end users (owners of prop planes and racing cars).

These cases appear to have relied on a qualitative weighing of the Hicks-Marshall conditions, rather than any quantitative analysis that often is used in critical loss/critical elasticity of demand. As shown below, much more systematic analyses can be done even if the key Hicks-Marshall parameters cannot be precisely estimated.

IV) USING HICKS-MARSHALL CONDITIONS TO INFORM ANTITRUST ANALYSIS OF INPUTS

A. Theory

34 Associated Octel Company Ltd. *et al.*, 125  F.T.C. 1286 (1998)(final order)
35 They also could not produce higher octane refinery streams from crude oil through various alternative processes, such as catalytic cracking and alkylation, or blend in other petrochemicals such as toluene or xylenes to achieve the desired octane levels. For additional detail on gasoline octane enhancers, see SRI Consulting, *Chemicals Economics Handbook*, “Gasoline Octane Improvers/Oxygenates,” April 2006 at 543.7500G to T. *Chemicals Economics Handbook* is an essential resource for the antitrust analyst of chemicals markets.
As discussed above, market definition analysis often compares an estimation the “critical elasticity” demand to an estimate of the actual elasticity of demand. For inputs, the elasticity of demand can often be assessed using the “Hicks-Marshall” conditions. Combining the “Hicks-Marshall” conditions of derived demand with critical elasticity of demand can provide important insights on relevant market definition for intermediate goods.

To make the Hicks-Marshall conditions in equation (1) more tractable for market definition, first recognize that in practice switching to other inputs will not be substantial enough to cause an increase in the price of these other inputs. Moreover, the Merger Guidelines state that the prices of other products should be assumed to remain constant when conducting a relevant market definition exercise. This implies that the supply elasticity of the other input ($n_i$) is infinite. In this case, equation (1) can be simplified as follows (once again, with elasticities stated in absolute value terms):

$$\varepsilon_i = n_i E_f + (1 - n_i) S_i \quad (2)$$

The “break-even” elasticity of demand below which a candidate product or set of products constitutes a relevant market (assuming linear demand) is equal to:

$$\varepsilon^* = \frac{1}{(M + t)} \quad (3)$$

Substituting equation (3) into equation (2) above and solving for the elasticity of demand for the downstream product yields a breakeven elasticity for the downstream product (as opposed to that for the input itself):

$$E_f^* = \frac{S_i (Mv_i + t v_i - M - t) + 1}{v_i (M + t)} \quad (4)$$

Product $i$ satisfies the conditions for constituting a relevant product market under break-even analysis when the actual elasticity of demand for the downstream product incorporating input $i$ falls below this level.

---

36 Merger Guidelines at Section 1.11, footnote 10.
37 Similar equations are readily adapted for the alternative assumptions of a constant elasticity of demand. For the relevant breakeven and critical elasticities of demand under the assumption of constant elasticity of demand, see Gregory Werden, Demand Elasticities in Antitrust Analysis, 66 Antitrust Law Journal 363, 389 (1998).
The equations for the “profit-maximizing” critical elasticity of demand are similar but distinct. The “profit-maximizing” critical elasticity of demand (assuming linear demand) for product $i$ is equal to:

$$
\varepsilon_i^* = \frac{1}{(M + 2t)} \quad (5)
$$

Substituting equation (5) into equation (2) above and solving for the final elasticity of demand yields the “profit-maximizing” critical elasticity of demand for the downstream product:

$$
E_f^* = \frac{S_i(Mv_i + 2tv_i - M - 2t) + 1}{v_i(M + 2t)} \quad (6)
$$

It is reasonable to expect that, in some cases, it may not be feasible for producers to switch from the input under consideration to an alternative input. In this case, the elasticity of substitution is zero and the price of other inputs will not change as a result of a price increase from the input at issue. In this case, equations (4) and (6) above simplify to:

$$
E_f^* = \frac{1}{v_i(M + t)} \quad (7)
$$

for the break-even elasticity of demand and

$$
E_f^* = \frac{1}{v_i(M + 2t)} \quad (8)
$$

for the profit-maximizing critical elasticity of demand.

B. Implications

Equations (4), (6), (7) and (8) demonstrate that solely examining the demand substitutability at the level of the customers using the input at issue can lead to the wrong delineation of the relevant market. For example, in equations (7) and (8) it is assumed that the customer of the input has no ability to substitute away from input $i$. Concluding that input $i$ must therefore constitute a relevant product market is mistaken, however. If elasticity of demand for the downstream product is above the critical (or break-even) level the candidate market does not constitute a relevant product market under critical
elasticity of demand analysis. Substitution by buyers downstream from the customers of the input can prevent a substantial price increase.

Combining critical loss/critical elasticity of demand analysis and the Hicks-Marshall conditions, one can define the relevant market using information about the elasticity of demand for the downstream product even when little or no direct information is available for the elasticity of demand for the input. Using information from the downstream market in some circumstances may also provide an alternative critical elasticity estimate and a “reality check” on critical loss analysis performed using only direct information on the input at issue.

As an example, consistent with the decisions in Goodrich and Octel involving PVC, assume that there are no reasonable substitutes for PVC in the production of vinyl products. Also consistent with Goodrich and Octel, assume that there are alternative end use products that do not employ PVC which could conceivably provide strong enough competition to prevent PVC from constituting a relevant product market. If, say, the gross margin percentage for PVC is 65%, PVC constitutes 5% of the total costs of the vinyl products, and one assumes an infinite elasticity of supply for other inputs, then the absolute value of the “break-even” elasticity of demand for vinyl products is very high, 28, using the typical 5% market test. Thus, the FTC’s conclusion that the elasticity of demand for vinyl products was “low” without an estimation of this elasticity would presumably be sufficient to conclude that PVC constituted a relevant product market.

More generally, when the elasticity of substitution is equal to zero, equations (7) and (8) above demonstrate that an input may not constitute a relevant market even when direct customers of that input have no ability to switch to alternative inputs. Examination of the range of the range of typical parameter values indicates that this is likely to occur only when the input at issue constitutes a fairly substantial portion of the total cost of the good incorporating that input. Price-cost margins are often in the range of 40-70 percent. In addition, estimated demand elasticities for downstream products have typically been below 5 in absolute value. Using a 40-70 percent range of price cost

---

39 [[Citation to be added]]
margins in equation (7) above, the critical elasticity of demand for the downstream product will be over 5 and likely to constitute a relevant product market under critical loss analysis -- unless the input constitutes 30% or more of the cost of the downstream product.40

When the elasticity of substitution is non-zero, the critical elasticity of demand of the downstream product depends on it, as well as on variable profit margins and the share of the input in total costs (as shown in equations (4) and (6) above). From these equations, the critical elasticity of final demand is a strictly decreasing function of $M$ and $S$ whereas $E^*_f$ is decreasing in $v_i$ only if certain conditions hold.41

While calculating $M$ and $v$ in practice may often be feasible, it may be difficult to estimate the elasticity of substitution and the elasticity of the final demand. However, with regard to the elasticity of final demand, the critical elasticity of final demand may be sufficiently high so that precisely calculating its value may be unnecessary. For example, assume that $M$ is equal to 0.4 and $v$ is equal to 0.3. If one were to estimate the elasticity of substitution at say, 2, then any value of final elasticity of demand below approximately 3 would indicate that input $i$ constitutes a relevant market (with a value lying above this indicating that it did not constitute a relevant market).42 There would be no need to precisely estimate this elasticity as one would only need determine if it is above or below

---

40 The critical elasticity of demand will exceed 10 unless the input constitutes at least 15% of the cost of the downstream product.

41 The proof for Eq. 4 is as follows:

$$E^*_f = \frac{S(Mv_i + tv_i - M - t) + 1}{v_i(M + t)} = \frac{S(M + t)(v_i - 1) + 1}{v_i(M + t)}.$$ 

Given that $v_i(M + t)$ is positive and $(v_i - 1)$ is negative, it is apparent that $E^*_f$ is decreasing in $S$.

Also further rearranging yields $E^*_f = \frac{S(v_i - 1)}{v_i} + \frac{1}{v_i(M + t)}$.

Examination of the second term in this equation shows that $E^*_f$ is decreasing in $M$ as both $v_i$ and $t$ are positive.

The last equation further reduces to:

$$E^*_f = \frac{S(v_i - 1)}{v_i} + \frac{1}{v_i(M + t)} = \frac{S_i - S_i}{v_i(M + t)} + \frac{1}{v_i(M + t)} = S_i + \frac{1 - SM - St}{v_i(M + t)}.$$ 

This equation demonstrates that $E^*_f$ is decreasing in $v_i$ only if the term $[1 - SM - St]$ is positive.

42 Using the breakeven critical elasticity. Similar examples can be shown for the profit maximizing elasticity as well.
3. Figure 1 shows in this example the different combinations of the elasticity of substitution and the breakeven elasticity of demand for the downstream good.

In other cases, the values of $M$, $S$, and $v$ may establish an $E^*_f$ that is so high or low that the relevant market question can be answered without estimating the final elasticity of demand. For example, as explained above, when the elasticity of substitution is very low (or equal to zero), $E^*_f$ will typically be very high unless the input at issue constitutes a relatively high proportion of the total costs of the downstream product. Moreover, from equations (4) and (6) above it can be seen that the Hicks Marshall conditions can yield a negative value of $E^*_f$, meaning that input $i$ does not constitute a relevant market regardless of the value of the final elasticity of demand. This can occur when the elasticity of substitution is high enough to establish that enough switching will occur among direct customers of the input to make a price increase unprofitable regardless of
the elasticity of final demand. For example, at an elasticity of substitution equal to 2 (in absolute value), a value of $v$ equal to .2, and a value of $M$ equal to .7, $E_f^*$ is negative (from equation (4) above).

As in the example above (and as depicted in Figure 1), suppose that $v$ is equal to 0.3 and $M$ is equal to 0.4. Assume that the estimated final elasticity of demand is equal to 3. In this case, if the elasticity of substitution is less than approximately 2, then input $i$ constitutes a relevant market (and it does not constitute a relevant market otherwise).

There is no need to precisely estimate the elasticity of substitution if its value can be placed at above or below 2. Moreover, just as in the case of estimating the elasticity of final demand, there are some values of the parameters in equations (4) and (6) that make estimation of the elasticity of substitution unnecessary. For example, if $v$ is equal to 0.8, $M$ is equal to 0.6, and the elasticity of demand for the downstream product is equal to 2, then input $i$ does not constitute a relevant market under critical loss analysis even if the elasticity of substitution is equal to zero.

To further assess the importance of precise estimation of $E_f^*$ or the parameters $E_f^*$ is a function of, we conducted a Monte Carlo style simulation (10,000 draws) of the values of $E_f^*$ that result from equation (4) under the assumption of a uniform probability distribution for $S$, $v$ and $M$ in which $S$ is assumed to be between 0 and 4, $v$ is assumed to lie between .001 and .8 and $M$ is assumed to lie between 0.4 and 0.7.43 Figure 2, below, shows the average break-even final elasticities resulting from different assumed ranges for $S$, $v$, and $M$. As can be seen in Figures 2A and 2B, relatively high values for $S$ (above 2.5) and relatively high values for $v$ (above 0.5) tend to result in low average values for $E_f^*$. As shown, when such values of either $S$ or $v$ prevail, $E_f^*$ is on average quite low meaning that the input does not constitute a relevant product market under critical loss analysis regardless of the value of the elasticity of demand for the downstream product.

Figure 3 below examines the conditions under which the knowledge of $M$ may assist with market definition at the input level. More specifically, Figure 3 shows the solutions for $E_f^*$ for different ranges of $M$, $v$, and $S$ (considering the ranges of two

---

43 In order to ensure consistency between the assumed parameters the following constraint was added $M<1/[(1-v)*S$. This ensures that the elasticity of the industry demand for the input is no greater than the implied elasticity of demand for individual suppliers of that input (as implied by the Lerner Index). Although, as shown above, $E_f^*$ is a decreasing function of $M$, the simulations do not show this pattern due in part to this constraint.
parameters at a time). Panels 1A, 1B, and 2 show that in addition to the already stated situations, i.e. relatively high values for $S$ (above 2.5) or relatively high values for $\nu$ (above 0.5), there are certain ranges for $M$ that when combined with information about the range of either $S$ or $\nu$, can lead to relevant market conclusions using critical loss analysis even with little or no information about the elasticity of final demand. In Panel 1A we have highlighted the instances where the average $E^*_{\gamma}$ is less than or equal to two and thus may serve as an indication that the intermediate good in question likely does not constitute a relevant market. Panel 1B corroborates these findings by highlighting all instances where the maximum $E^*_{\gamma}$ is less than 2.0. Panel 2 in Figure 3 examines pairings for $M$ and $\nu$ resulting in relatively high average values for $E^*_{\gamma}$ (we have highlighted values greater than 10) where one will often accept the hypothesis of a relevant market using critical loss analysis. Panel 3 focuses on the variations in $E^*_{\gamma}$ in response to different values for $S$ and $\nu$ and provides additional ranges for the two parameters that result in very high or very low values for the critical demand elasticity of the downstream product.

Figures 2 and 3 illustrate that it may not be necessary to have full knowledge of and/or precisely estimate all of the parameters entering the demand system for the intermediate product to be able to address the market definition question in a satisfactory manner. As shown above there are many ranges of the parameters $S$, $\nu$, and $M$ that would assist with market definition--even in the absence of precise estimates for the demand elasticity of the downstream product.

As can be seen from these examples, a detailed analysis of input market definition can be done by applying the Hicks-Marshall conditions, even when there are not exact estimates of the key parameters. Moreover, this systematic approach is likely to yield a substantially more reliable market definition analysis than the qualitative analyses that have been done in past cases.
Figure 2

Figure 2A: Simple Average of $E^*$, by Range of $S$

Figure 2B: Simple Average of $E^*$, by Range of $v_i$

Figure 2C: Simple Average of $E^*$, by Range of $M$

Notes:
1. Solutions subject to the following constraint: $M < 1/(1 - \nu)S$ ensuring that the elasticity of the industry demand for the input is no greater than the implied elasticity of demand for individual suppliers of that input (as implied by the Lerner Index).
2. Values for $S$, $\nu$ and $M$ are drawn from a uniform distribution in which $S$ is assumed to be between 0 and 4, $\nu$ is assumed to lie between .001 and .8 and $M$ is assumed to lie between .4 and .7.
Draft – Please Do Not Circulate Without Permission of Authors

Figure 3
Simulated Solutions\(^1\) for \(E^* \gamma\) : Joint Distributions of M, S and v
Based on a Monte Carlo Simulation of 10,000 Draws\(^2\)

Panel 1A: Simple Average of \(E^* \gamma\) by joint distribution of M and S
[Highlighted cells show instances where the average \(E^* \gamma\) is less than 2]

<table>
<thead>
<tr>
<th>Range for M</th>
<th>Range for S</th>
</tr>
</thead>
<tbody>
<tr>
<td>M: [0.4; 0.45)</td>
<td>S: [0; 0.5)</td>
</tr>
<tr>
<td>[Highlighted cells show instances where the average (E^* \gamma) is less than 2]</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>Overall</td>
</tr>
</tbody>
</table>

Panel 1B: Maximum of \(E^* \gamma\) by joint distribution of M and S
[Highlighted cells show instances where the maximum \(E^* \gamma\) is less than 2]

<table>
<thead>
<tr>
<th>Range for M</th>
<th>Range for S</th>
</tr>
</thead>
<tbody>
<tr>
<td>M: [0.4; 0.45)</td>
<td>S: [0; 0.5)</td>
</tr>
<tr>
<td>[Highlighted cells show instances where the maximum (E^* \gamma) is less than 2]</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>Overall</td>
</tr>
</tbody>
</table>

Panel 2: Simple Average of \(E^* \gamma\) by joint distribution of M and v
[Highlighted cells show instances where the average \(E^* \gamma\) is greater than 10]

<table>
<thead>
<tr>
<th>Range for M</th>
<th>Range for v</th>
</tr>
</thead>
<tbody>
<tr>
<td>M: [0.4; 0.45)</td>
<td>v: [0.001; 0.1)</td>
</tr>
<tr>
<td>[Highlighted cells show instances where the average (E^* \gamma) is greater than 10]</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>Overall</td>
</tr>
</tbody>
</table>

Panel 3: Simple Average of \(E^* \gamma\) by joint distribution of S and v
[Highlighted cells show instances where the average \(E^* \gamma\) is greater than 10 or less than 2]

<table>
<thead>
<tr>
<th>Range for v</th>
<th>Range for S</th>
</tr>
</thead>
<tbody>
<tr>
<td>v: [0.001; 0.1)</td>
<td>S: [0; 0.5)</td>
</tr>
<tr>
<td>[Highlighted cells show instances where the average (E^* \gamma) is greater than 10 or less than 2]</td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>Overall</td>
</tr>
</tbody>
</table>

Notes:
1. Solutions subject to the following constraint: \(M \leq \frac{1}{1(E^* \gamma)S}\) ensuring that the elasticity of the industry demand for the input is no greater than the implied elasticity of demand for individual suppliers of that input (as implied by the Lerner Index).
2. Values for S, v and M are drawn from a uniform distribution in which S is assumed to be between 0 and 4, v is assumed to lie between 0.001 and 0.8 and M is assumed to lie between 0.4 and 0.7.
V. CONCLUSION

The question of the appropriate definition of the relevant market for an input has often been an important one in antitrust inquiries. As established by examining the tools of relevant market definition in light of the long established Hicks-Marshall conditions of derived demand for an input, downstream conditions can be very important in properly delineating the relevant market for inputs. Unfortunately, previous analyses have differed on whether or not these conditions have been examined.

In this article, we adapt the often used tool of critical loss analysis to take into account the Hicks-Marshall conditions of derived demand for inputs. Several important insights emerge. First, solely examining the demand substitution possibilities for direct consumers of an input can lead to the wrong relevant market definition. Second, critical loss analysis in conjunction with the Hicks-Marshall conditions can provide an additional avenue for delineating the relevant market when it is difficult to directly estimate the elasticity of demand for an input. They can also provide a “reality check” on the results obtained from typical critical loss analysis. Third, reasonable ranges on parameters related to the ability of users of the input to substitute to other inputs and downstream market conditions as opposed to precise estimation will often be sufficient to define relevant markets for inputs using critical loss analysis in practice, and these analyses should be more economically reliable than qualitative analyses that have been done in past cases.